CORRECTION TO "PROJECTIVE PLANES"[1]

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It has been brought to my attention by Professor M. F. Smiley that there is an error in Theorem 6.4. Equations 6.11, 12, 13, 14, 15, 16 on page 268 have been derived from an application of Theorem L with A, B, M, N on $y=0$, and not on $x=0$ as needed for Theorem 6.4.

We may instead apply Theorem L with A, B, M, N on $x=0$ taking $A=(0,0)$, $B=A$ ($x=$const.), $X=B$ ($y=$const.), $M=(0, 1+a)$, $N=(0, b+ab)$, $R=(-1-a, 0)$ $Y=(1, 1+a)$ with a, b arbitrary. Then ASY is $y=x(1+a)$; RSM is $y=x+a+1$; BYZ is $x=1$; NZX is $y=b+ab$. Here $Z=(1, b+ab)$ and $S=(d, d+a+1)$ where $d+a+1=d(1+a)$. Then RTN is $y=xb+b+ab$ and ATZ is $y=x(b+ab)$. Hence T is $(e, e(b+ab))$ where $e(b+ab)=eb+b+ab$. From the collinearity of BST, we conclude $d=e$. Thus we have

$$d+a+1=d(1+a),\ d(b+ab)=db+b+ab.$$

If we put $d=u+1$ we find

$$ua=1,\ u(ab)=b$$

for any a, b and an appropriate u. In particular if b is chosen so that $ab=1$, then $b=u$ and so from $ua=1$ we derive $au=1$, whence we may write $u=a^{-1}$ with (6.13) $a^{-1}a=1$, $aa^{-1}=1$. We now have also for arbitrary a, b, $a^{-1}(ab)=b$. Thus applying Theorem L with A, B, M, N on $x=0$ as well as on L_∞ we have all laws of (6.17) except $(ba)a^{-1}=b$, and the law (6.14) $(br)^{-1}=r^{-1}b^{-1}$. With these laws collineation (δ) on page 270 is valid, and from Theorem 6.3 we now conclude that Theorem L holds for all choices of A, B, M, N on L_∞ or $x=0$. Since the validity of collineation (α) on page 270 depends solely on these laws, it follows that Theorem L is valid for A,B,M,N on any line $x=c$, of the pencil through point A. Any line not in this pencil may be taken as the line $y=0$ and the application of Theorem L on $y=0$ produces (6.14) whence from $a(a^{-1}b^{-1})=b^{-1}$ we derive $b=(ba)a^{-1}$ and the rest of Theorem 6.4 holds. In the light of these observations the statement of Theorem 6.4 should be altered as follows:

THEOREM 6.4. *If Theorem* L *is satisfied for all choices of A, B, M, N on two lines in a plane, then it is satisfied for all choices of A, B, M, N on any line of the pencil through the intersection of these two. If the two lines are taken as L_∞ and $x=0$, then the natural ring of π satisfies*

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$$
\begin{array}{lll}
& a \cdot m \circ b = am + b, & a(b+c) = ab + ac, \\
& (a+b)+c = a+(b+c), & (a+b)m = am + bm, \\
(6.17.1) & a + b = b + a, & a1 = 1a = a, \\
& a + 0 = 0 + a = a, & aa^{-1} = a^{-1}a = 1,\ a \neq 0, \\
& a + (-a) = (-a) + a = 0, & a^{-1}(ab) = b.
\end{array}
$$

If Theorem L *holds for* A, B, M, N *on three lines not in a pencil, then it is a universal theorem in* π. *In addition to* (6.17.1) *we also have* (6.17.2) $(ab)^{-1} = b^{-1}a^{-1}$, $(ba)a^{-1} = b$ *and any natural ring of* π *is an alternative field. The collineation group of* π *is transitive on the triangles of* π.

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Don Mittleman, *The unions of trajectorial series of lineal elements generated by the plane motion of a rigid body.*

p. 503, line 2 of Theorem 1. For "(x, y)" read "$u(x, y)$."